PROM^2 for Girls 2025
Individual Round Solutions

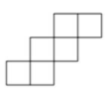
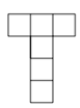
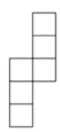
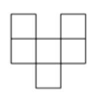
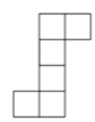
PRISMS Math Team

April 2025

Problem 1
Evaluate $2025 - (20 + 25)^2$.
(A) 0 (B) 1 (C) 25 (D) 425 (E) 800

Answer: A
Solution:
$2025 - (20 + 25)^2 = 2025 - 45^2 = 2025 - 2025 = \boxed{0}$.
Credit: Joseph Li

Problem 2
Which of the following patterns below could not be unfolded from a cube?

(A) 
(B) 
(C) 
(D) 
(E) 

Answer: D
Solution:
Option $\boxed{\text{D}}$ cannot be unfolded from a cube.
Credit: Jessie Wang

Problem 3
Anna thought of four distinct positive integers. The product of the smallest and largest numbers is 32. The product of the two remaining numbers is 14. What is the sum of all four numbers?
(A) 27 (B) 33 (C) 42 (D) 46 (E) 47

Answer: C
Solution:

Let the numbers be $a < b < c < d$ with $ad = 32$, $bc = 14$. As $b > a \geq 1$, $b \geq 2$. Since $b < c$ and $bc = 14$, we must have $b = 2$ and $c = 7$. Then $a = 1$, so $ad = 32$ gives $d = 32$. Summing, we get $1 + 2 + 7 + 32 = \boxed{42}$.
Credit: Anna Fenchenko

Problem 4
A string of letters is called an interesting word if it contains the five letters P, R, O, M, and M in some order, and only those letters. How many interesting words exist?
(A) 1 (B) 30 (C) 32 (D) 60 (E) 120

Answer: D
Solution:
The number of permutations of {P, R, O, M, M} is $\frac{5!}{2!} = \boxed{60}$.
Credit: Charlie Gu

Problem 5
A drawer contains 5 red socks, 7 blue socks, and 9 green socks. What is the minimum number of socks you must pull to guarantee two matching pairs?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Answer: C
Solution:
When we pull out 3 red, 1 blue, and 1 green socks, we don't have two matching pairs. Thus, the answer should be greater than 5.
f we pull out $\boxed{6}$ socks, from the Pigeonhole Principle, 2 of them have the same color, which form one matching pair.
Now we have 4 socks remaining, still because of the Pigeonhole Principle, 2 of them have the same color, which form the 2nd matching pair.
Credit: Ivey Wang

Problem 6
Find the remainder when 2025^{25} is divided by 125.
(A) 0 (B) 1 (C) 5 (D) 25 (E) 100

Answer: A
Solution:
Because 2025 is divisible by 5, 2025^{25} is divisible by 5^{25}, which is clearly divisible by $5^3 = 125$ and has remainder $\boxed{0}$.
Credit: Joseph Li

Problem 7
Joker has several identical rectangular cards. If he attaches 2 cards together as shown below, the perimeter of the shape is 56 cm; if he attaches 3 cards together, the perimeter is 72 cm. What is the perimeter of one single card?

(A) 24 (B) 28 (C) 32 (D) 36 (E) 40

Answer: E
Solution:
Denote the length and width of each rectangular card by a and b. Compare the perimeters of the two rectangles. The perimeter of the rectangle on the right is greater than that of the rectangle on the left by twice the width of the small rectangle. Thus, $2b = 72 - 56 \implies b = 8$.
From the first rectangle, we know that $2a + 4b = 56$, so we get $a = 12$. This gives the desired perimeter of $2(a + b) = \boxed{40}$ cm.
Credit: Joseph Li

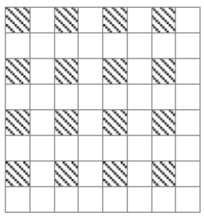
Problem 8
Let a and b be positive integers. If the greatest common divisor of a and b is 12, and the least common multiple of a and b is 720, what is the minimum possible value of $a + b$?
(A) 192 (B) 204 (C) 216 (D) 228 (E) 240

Answer: B
Solution:
Suppose $a = 12c$ and $b = 12d$, then $\gcd(c, d) = 1$. The least common multiple of a and b is $12cd = 720$, so $cd = 60$. It is commonly known that for a fixed product of two positive numbers, the smaller their difference, the smaller their sum. (This can be proved by the identity $(c + d)^2 - (c - d)^2 = 4cd$).
Therefore, to minimize the value of $a + b = 12(c + d)$, we need to minimize the difference between c and d. As $cd = 60$, the closest pair is 6 and 10. However, they are not co-prime. Continuing, the next pair is 5 and 12 which gives us the minimum possible value of $a + b = 12(c + d) = 12(5 + 12) = \boxed{204}$.
Credit: Rafi Shang

Problem 9
Alan is the leader of the PRISMS Chess Club. What is the maximum possible

number of kings that Alan may place on an 8×8 chessboard such that no two kings are in neighboring squares? (Two squares are neighbors if they share a common edge or a common vertex.)
(A) 16 (B) 17 (C) 18 (D) 30 (E) 32

Answer: A
Solution:



We can divide the 8×8 chessboard into 16 2×2 squares. In each 2×2 square, any two unit squares are neighbors. Thus, we can place at most 1 king in each 2×2 square. This implies that we can place at most 16 kings on the entire chessboard. As shown above, we can place 16 kings in the colored squares such that no two kings are in neighboring squares. So, the desired answer is $\boxed{16}$.
Credit: Terry Huang

Problem 10
PRISMS students are actively involved in community service. The PRISMS Math Club regularly hosts fun math activities for young children at the Princeton Public Library. In this month's event, they tried sorting kids into groups. When putting them into groups of 6, there were 4 kids left over. When trying groups of 7, 3 kids still wouldn't fit. Which of the following could be the number of kids in that event?
(A) 31 (B) 39 (C) 46 (D) 52 (E) 60

Answer: D
Solution:
Suppose there are n kids in that event. The condition tells us that when n is divided by 6, the remainder is 4, and when n is divided by 7, the remainder is 3. Among all the options, the only number that satisfies both conditions is $\boxed{52}$.
Note: all the numbers in the form $42m + 10$ satisfy the conditions.
Credit: Jessie Wang

Problem 11
Minnie, Daisy, and Clarabelle are playing a game: each of them is either telling the truth or telling a lie.

- Minnie says, “Daisy tells a lie.”
- Daisy says, “Clarabelle tells the truth.”
- Clarabelle says, “Minnie tells a lie.”

If at most one of them tells a lie, who is the liar?
(A) Minnie (B) Daisy (C) Clarabelle (D) Nobody (E) Can’t determine

Answer: A
Solution:
Because at most one of them tells a lie, what Minnie and Clarabelle say cannot be both true. Thus, Daisy tells the truth, which also implies Clarabelle tells the truth. Therefore, $\boxed{\text{Minnie}}$ tells a lie based on what Clarabelle says.
Credit: Rafi Shang

Problem 12
David found that he could write the same positive integer into each square in the following equation to make both sides equal:

$$25 \times \square - (\square \times \square + 20 \times \square) \div \square = 700$$

What is the integer David found?
(A) 28 (B) 30 (C) 35 (D) 140 (E) 175

Answer: B
Solution:
Denote the integer David found by n: then the left side of the equation is

$$25n - (n^2 + 20n) \div n = 25n - (n + 20) = 24n - 20.$$

From $24n - 20 = 700$, we get that $n = \boxed{30}$.
Credit: Joseph Li

Problem 13
During a community volunteer event, Allen the Elf needed to help pack greeting cards into envelopes. By 8:40 am, he had packed 90 envelopes. By 10:10 am, he had packed a total of 360 envelopes. Allen packs envelopes at a constant pace. At 9:30 am, Terry the Reindeer had come and taken away all the envelopes that Allen had packed by that time. How many envelopes did Terry take?
(A) 195 (B) 210 (C) 225 (D) 240 (E) 255

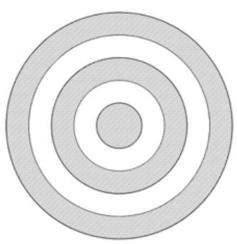
Answer: D
Solution:

There are 90 minutes from 8:40 am to 10:10 am, so Allen packs $(360-90)\div 90 = 3$ envelopes in one minute. From 8:40 am to 9:30 am, there are 50 minutes. Thus, Terry took $90 + 3 \times 50 = \boxed{240}$ envelopes.
Credit: Joseph Li

Problem 14
Let $d_1 > d_2 > d_3 > ... > d_n$ be all the positive divisors of 2025. Find the value of $d_2 - d_3$.
(A) 2 (B) 180 (C) 270 (D) 450 (E) 1350

Answer: C
Solution:
We just need to realize that $d_1 d_n = d_2 d_{n-1} = d_3 d_{n-2} = ... = 2025$. Because it is easier to find the smallest divisors of 2025, we get that $d_2 = \frac{2025}{d_{n-1}} = \frac{2025}{3} = 675$ and $d_3 = \frac{2025}{d_{n-2}} = \frac{2025}{5} = 405$. Finally, $d_2 - d_3 = 675 - 405 = \boxed{270}$.
Credit: Joseph Li

Problem 15
The figure shows five concentric circles with radii 1, 2, 3, 4, and 5 units, respectively. The shaded regions form alternating rings between the circles, starting with the innermost circle being shaded. Calculate the total area of the shaded regions.



(A) 10π (B) 15π (C) 16π (D) 25π (E) 30π

Answer: B
Solution:
The desired area is $(\pi \cdot 5^2 - \pi \cdot 4^2) + (\pi \cdot 3^2 - \pi \cdot 2^2) + \pi \cdot 1^2 = \boxed{15\pi}$.
Credit: Rafi Shang

Problem 16
The 6-digit number, $\overline{a2025b}$, is a multiple of 99. What is the value of $a+b+a\times b$?

(A) 17 (B) 23 (C) 27 (D) 29 (E) 35

Answer: D
Solution:
We have that $\overline{a2025b} = \overline{a2} \cdot 10^4 + 2 \cdot 10^2 + \overline{5b} = \overline{a2} \cdot (9999+1) + 2 \cdot (99+1) + \overline{5b}$. So considering mod 99, we have $\overline{a2025b} = \overline{a2} + 2 + \overline{5b} = 10a + b + 54 \mod 99$. Because $0 \le a, b \le 9$ and $10a + b + 54 = 99$, we can get that $a = 4$ and $b = 5$. Thus, $a + b + a \times b = \boxed{29}$.
Credit: Kris Sheng

Problem 17
If $x + \frac{1}{x} = 3$, what is the value of $x^3 + \frac{1}{x^3}$?
(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

Answer: B
Solution:
To make the exponents 3, we try $(x + \frac{1}{x})^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$. Now we can see that $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3(x + \frac{1}{x}) = 3^3 - 3 \cdot 3 = \boxed{18}$.
Credit: Ivey Wang

Problem 18
Sophia qualified for MOP (Mathematical Olympiad Program) last year. Starting from day n, Sophia practiced geometry problems. She solved n geometry problems on that day, and she always solved one more problem than she did on the previous day (so she solved $n+1$ problems on the next day). After she solved $n+10$ problems on day $n+10$, she decided to focus on algebra and didn't practice geometry anymore. If she solved 165 geometry problems across those days, what is the value of n?
(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

Answer: B
Solution:
There are 11 days from day n to day $n + 10$. On the middle day, which is day $n + 5$, Sophia solved $165 \div 11 = 15$ geometry problems. Thus, on day n, she solved $15 - 5 = \boxed{10}$ problems.
Credit: Terry Huang

Problem 19
Sigma is a PRISMS student who has a wide range of interests. He regularly

attends 3 clubs: History, Linguistics and Math. He attends History Club every 4 days, Linguistics Club every 5 days and Math Club every 6 days. On January 1st, 2025, he attended all 3 clubs. On how many days in May of 2025 he will attend at least 2 clubs?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

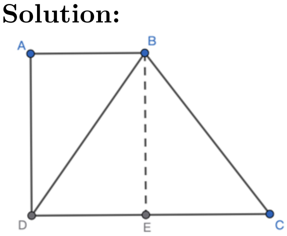
Answer: B
Solution:
The least common multiple of 4, 5, and 6 is 60, so Sigma attends all 3 clubs on the same day after 60 days. Because there are 31+28+31+30=120 days from January to April, Sigma attends all 3 clubs on May 1st, 2025.

- The least common multiple of 4 and 5 is 20, so Sigma attends History and Linguistics on May 21st, 2025.
- The least common multiple of 5 and 6 is 30, so Sigma attends Linguistics and Math on May 31st, 2025.
- The least common multiple of 4 and 6 is 12, so Sigma attends History and math on May 13th and 25th, 2025.

In total, Sigma will attend at least 2 clubs on $\boxed{5}$ of the days.
Credit: Charlie Gu

Problem 20
In trapezoid $ABCD$, $AB \parallel CD$, $AB = 5$, $BD = 13$, $CD = 10$, $AD = 12$. Find the length of BC.
(A) 6 (B) 9 (C) 10 (D) 12 (E) 13

Answer: E
Solution:



As $5^2 + 12^2 = 13^2$, by Pythagorean theorem, triangle ABD is a right triangle. Let E be the foot of the perpendicular from B to side CD. Then $ABED$ is a rectangle with $DE = AB = 5$ and $BE = AD = 12$. Because $CD = 10$, we get $CE = CD - DE = 5$. As angle BEC is right, using Pythagorean theorem again, $BC^2 = DE^2 + CE^2 = 5^2 + 12^2 = 169$. Thus, $BC = \boxed{13}$.
Credit: Ivey Wang

Problem 21

How many ways can you arrange the six little ponies, Applejack, Rainbow Dash, Twilight Sparkle, Pinkie Pie, Fluttershy, and Rarity in a row if Twilight Sparkle does not want to stand at the edge, and Pinkie Pie and Fluttershy want to stand next to each other?

(A) 72 (B) 144 (C) 160 (D) 192 (E) 200

Answer: B

Solution:

We can arrange the order by the following steps:

- Step 1: Consider Pinkie Pie and Fluttershy as one. We have 2 choices to decide the order of those two.
- Step 2: Now we arrange the order for five ponies (PP and F are considered as one now). We have 3 choices to put Twilight Sparkle at a non-edge position.
- Step 3: We arrange the order of other 4 ponies without any restriction. We have 4!=24 choices.

Overall, we have $2 \cdot 3 \cdot 24 = \boxed{144}$ ways to arrange those ponies.

Credit: Asel Alibekova

Problem 22

We have 100 lamps are labelled $1, 2, \ldots, 100$, all turned off at the beginning. At the first round, every lamp is turned on. For any $1 \leq n \leq 100$, at the n_{th} round, all the lamps that are labelled with a multiple of n change the status. After 100 rounds, how many lamps are turned on at the end?

(A) 0 (B) 10 (C) 50 (D) 90 (E) 100

Answer: B

Solution:

A lamp is turned on implies that it changes the status an odd number of times in 100 rounds. Because a lamp only changes its status when it is a multiple of the round number, a lamp is turned on at the end if and only if it has an odd number of divisors. Thus, it is a perfect square. From 1 to 100, there are $\boxed{10}$ perfect squares, which are $1^2, 2^2, \ldots, 10^2$.

Credit: Jessie Wang

Problem 23

There are 60 people that took turns speaking. The first three speakers said the

same thing: "I have always told the truth!" The following 57 speakers also said the same phrases: "Among the previous three speakers, exactly two told the truth." What is the maximum possible number of speakers who told the truth?
(A) 20 (B) 30 (C) 40 (D) 45 (E) 60

Answer: D
Solution:
First, the first four speakers could not be all truth tellers. Otherwise, for the 4th speaker, 3 of the previous three speakers told the truth, which isn't possible (a contradiction)! Next, let's consider 4 consecutive speakers, $4n+1$, $4n+2$, $4n+3$, and $4n+4$. Similarly, these four speakers could not be all truth tellers. Otherwise, the speaker $4n+4$ should have said that all previous three speakers told the truth. The 60 people can be split into 15 groups of 4 and each group has at most 3 truth tellers. Thus, there are at most $3 \cdot 15 = 45$ truths told among them. Also, by having all the 4th, 8th, 12th,..., 60th speakers each tell a lie while all others tell the truth, we can achieve exactly $\boxed{45}$ speakers telling the truth.
Credit: Anna Fenchenko

Problem 24
A regular 18-gon and a regular 24-gon are inscribed in the same circle. A point is common if it lies on both the boundaries of two polygons. Denote m as the minimum possible number of common points, and n as the maximum possible number of common points. Find the value of $m+n$.
(A) 66 (B) 69 (C) 72 (D) 75 (E) 78

Answer: A
Solution:
Obviously, the side length of the regular 24-gon is shorter than the side length of the regular 18-gon. For a fixed side of the 18-gon, AB, there is at least one vertex of the 24-gon lies on arc AB of the circle. Thus, side AB has exactly two common points with the 24-gon. For all 18 sides of the 18-gon, they have $2 \cdot 18 = 36$ common points in total. When they don't coincide with each other (we can make it happen by rotating the 24-gon by a small angle). So, we get the maximum number of common points, $n = 36$.
The number of the common points could also be less when common points on two sides coincides. This happens when they are both at one vertex of the 18-gon. Consider labelling the vertices of the 18-gon with $A_1, A_2, ...A_{18}$. If the first vertex A_1 of the 18-gon is also a vertex of the 24-gon, then A_4, A_7, A_10, A_13, A_16 are also vertices of the 24-gon, and each of them is counted twice as common points. This case gives us the minimum possible number of common points $m = 36 - 6 = 30$. Finishing the problem, we get $m + n = 36 + 30 = \boxed{66}$.
Credit: Sigma Liu

Problem 25
Mr. Nallbani, the popular art teacher at PRISMS, plans to color a window. For each of the 9 unit squares, he will choose one color from red, yellow, or green. For any two adjacent unit squares, two colors must be different (2 unit squares are adjacent if and only if they share one common edge). How many possible coloring results does Mr. Nallbani have?
(A) 6 (B) 48 (C) 108 (D) 246 (E) 294

Answer: D
Solution:
WLOG (Without Loss of Generality), suppose the center square is painted green and later we will multiply our answer by 3. Now consider the 4 squares that are adjacent to the center square (we will call them side squares), they can only be painted red or yellow. Next we will count the coloring results by cases.

- Case 1: if the 4 side squares are painted the same color. We have 2 choices for the color for the side squares. For each square at the corner (we will call them corner squares), we can use either the other color or green to paint it. In total, we have $2^5 = 32$ choices.

2	X	2
X	G	X
2	X	2

Case 1

2	X	1
X	G	Y
2	X	1

Case 2

2	X	1
X	G	Y
1	Y	2

Case 3

1	X	1
Y	G	Y
1	X	1

Case 4

- Case 2: If exactly 3 side squares are painted the same color. We have 2 choices for the color that is painted 3 times and 4 choices for which 3 side squares are painted that color. For each corner square, if its neighbors are painted different colors, it must be painted green. In total, we have $2 \cdot 4 \cdot 2 \cdot 2 = 32$ choices.

- Case 3: If 2 side squares are painted red and the other two yellow, and two red side squares are in adjacent sides, we have 4 choices for the 2 side squares that are red. Then consider the corner squares, it is easy to see that we have $4 \cdot 2 \cdot 2 = 16$ choices.

- Case 4: If 2 side squares are painted red and the other two yellow, and two red side squares are in opposite sides, we have 2 choices for the 2 side squares that are red. Then all corner squares must be painted green. In total, we have 2 choices.

Putting everything together, the total number of colorings are $3 \cdot (32+32+16+2) = \boxed{246}$.
Credit: Joseph Li