

PROM² for Girls 2022

March 12, 2022

3 points

1. Evaluate $2022 - 20 \times 22$.

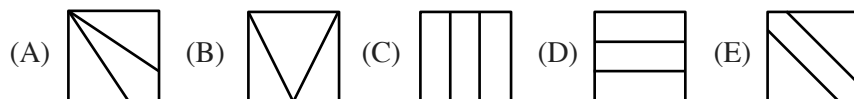
- (A) 0 (B) 1522 (C) 1582 (D) 1622 (E) 1682



2. How many times must 7 be added to -12 to get 44?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

3. Which of the following squares is not divided into three parts with equal areas?



4. How many integers between 100 and 999 (inclusive) are divisible by 37?

- (A) 21 (B) 22 (C) 23 (D) 24 (E) 25

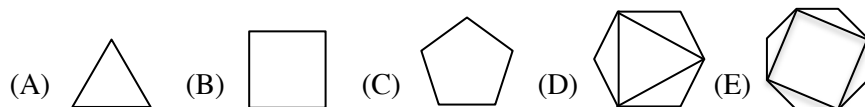
5. Evaluate $\frac{331}{3} - \frac{551}{5}$.

- (A) 0 (B) $\frac{1}{15}$ (C) $\frac{2}{15}$ (D) $\frac{4}{15}$ (E) $\frac{8}{15}$

*It is impossible to be
a mathematician
without being a poet
in soul.*

-Sofia Kovalevskaya

6. Which of the following shapes has the most lines of symmetry?



7. A rectangle has a width of 4 and an area of 64. What is the area of the square with the same perimeter as the rectangle?

- (A) 25 (B) 100 (C) 289 (D) 400 (E) 1156

8. On the first day of 2021, Mr. Li told all of the students at PRISMS, “2021 is the product of two consecutive prime numbers!” Which is true because $2021 = 43 \times 47$. He then proclaimed “And this is probably the only year in our lifetime that will have this property!” How many years will it take for there to be another year that is a product of two consecutive prime numbers?

- (A) 94 (B) 188 (C) 282 (D) 376 (E) 470

9. Trinity runs in a triathlon consisting of three sections: swimming, cycling, and running. She can swim at 3 miles per hour (mph), cycle at 15 mph, and run at 6 mph. If the swimming section is 1 mile long, the cycling section is 40 miles long, and the running section is 10 miles long, how many minutes will the triathlon take her in total?

- (A) 216 (B) 240 (C) 263 (D) 270 (E) 280

10. Seven children were seated at a round table, each with their own pile of candies. No two children sitting next to each other have the same number of candies. Find the minimum total number of candies that the seven children had. (Note: A pile of "candies" can be just 1 piece of candy.)

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

Reserve your right to think, for even to think wrongly is better than not to think at all.
----- Hypatia

4 points



11. As shown in the diagram, a square is divided into three identical rectangles. If the perimeter of each rectangle is 120, what is the perimeter of the square?
(A) 80 (B) 160 (C) 180 (D) 288 (E) 360

12. Daniella, Emma, and Valentine started packing the PROM² T-shirts and other materials for participants at 8:00 AM. By 8:05 AM, Daniella had packed 2 packages while Emma and Valentine had each packed 3 packages. At some time Helena joined the team and they were able to get all 300 packages packed by 10:30 AM. If Helena can pack 4 packages every 5 minutes, then what time did Helena join the team?
(A) 9:15 AM (B) 9:25 AM (C) 9:30 AM (D) 9:42 AM (E) 9:45 AM

13. The side lengths of a triangle are all prime numbers and the sum of the three lengths is 24. What is the maximum possible product of the three side lengths?
(A) 114 (B) 170 (C) 240 (D) 242 (E) 312

$$\begin{array}{r}
 P \ 1 \ 9 \\
 4 \ R \ 3 \\
 + \ M \ R \ 6 \ O \\
 \hline
 2 \ 5 \ M \ 6
 \end{array}$$

14. In the equation on the right, letters ‘P’, ‘R’, ‘O’, and ‘M’ represent distinct digits among 0 to 9. What is the sum of the 4 digits that the letters represent?
(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

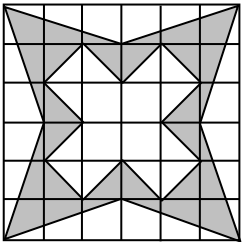
15. In the country of PRISM^{agica}, the Mathemagicians use three types of currency: *Matho*, *Magico*, and *Spello*. A *Magico* is worth 4 *Spellos*, and 5 *Magicos* is worth 6 *Spellos* and 7 *Mathos*. How many *Mathos* have the same value as 17 *Magicos* and 18 *Spellos*?
(A) 43 (B) 52 (C) 70 (D) 86 (E) 104

16. There are 12 PRISMS students taking the course ‘Industrial Design’. They used the 3D printers in the lab to make special badges for local kids. One of those 12 students, Angel, found that if she made 3 times as many badges as planned, then the average number of badges that each student made would be 7; and, if she made 7 times as many badges as planned, then the average number would be 13. How many badges did Angel plan to make?
(A) 10 (B) 12 (C) 15 (D) 18 (E) 20

17. Felicity has 82 cents in pennies and nickels. This morning her younger brother counted the money and said it was worth \$1.47. Felicity found out that her brother mistook all her nickels for dimes. How many pennies does Felicity have?
(A) 13 (B) 15 (C) 17 (D) 19 (E) 21

18. Lucy has 6 cards labeled from 1 to 6. She splits the cards into two piles of three. She then multiplies the sums of the cards in each pile. How many different possible products are there?
(A) 5 (B) 6 (C) 9 (D) 10 (E) 11

19. As shown on the right, 36 unit squares are assembled to form a 6 by 6 grid. What is the area of the shaded region?
(A) 14 (B) 15 (C) 16 (D) 18 (E) 19



20. The Doggy and the Piggy are counting numbers at the same speed. Which number will the Doggy and the Piggy speak out at the same time?



- (A) 277 (B) 287 (C) 297 (D) 307 (E) 317

5 points

21. Five students Anna, Belle, Cinderella, Dahlia, and Elsa sat around a round table (but not necessarily in that order). Mrs. D'Angelo distributed 5 cards labeled from 1 to 5 to each student.

Anna said: The difference between the numbers held by the two students next to me is 1.

Belle said: The difference between the numbers held by the two students next to me is 2.

Cinderella said: The difference between the numbers held by the two students next to me is 3.

Dahlia said: The difference between the numbers held by the two students next to me is 4.

What is the difference between the numbers held by the two students next to Elsa?

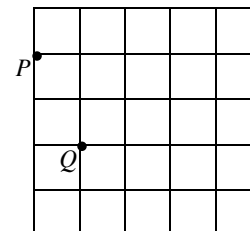
- (A) 1 (B) 2 (C) 3 (D) 4 (E) Cannot be determined

22. $10 + 9 + 8 + \dots + 2 + 1 = 55$

Karen changed some of the plus signs in the above equation to minus signs and found that the new result was 39. How many possible new equations exist?

- (A) 6 (B) 7 (C) 8 (D) 21 (E) 22

23. 25 unit squares form a 5 by 5 grid with 36 vertices. Joy wants to choose 3 points from those 36 vertices to form a triangle. She already chose P and Q . How many different vertices can the third point, R , be on such that the triangle PQR has an area that is a positive integer?

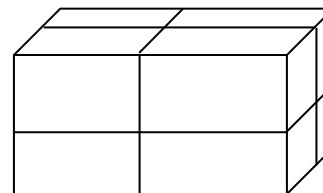


- (A) 4 (B) 5 (C) 15 (D) 16 (E) 18

24. Melinda and Estella go running on their school's 400-meter round track every afternoon. One day at 4:15 PM, Estella started running counterclockwise at a constant speed of 2.5 meters per second. At 4:20 PM, at the same starting point, Melinda started running clockwise at a constant speed of 3 meters per second. They both stopped running at 4:30 PM. How many times did they meet during the run?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

25. There is a cuboid which has been cut three times in different directions that are parallel to the faces. After the first cut, the sum of the surface areas of the two small cuboids is 472 square inches. After the second cut, the sum of the surface areas of the four smaller cuboids is 632 square inches. After the third cut, the sum of the surface areas of the eight smaller cuboids is 752 square inches. What is the area, in square inches, of the smallest face among the six faces of the original cuboid?



- (A) 48 (B) 60 (C) 80 (D) 96 (E) 120

26. PRISMS offers many interesting clubs to students. Christina couldn't decide which 3 clubs to choose from:

1. Choir 2. Robotics 3. Yearbook 4. Rock Band 5. Theater

Each of her roommates gave her two suggestions:

Cici: "Choose the Rock Band Club" and "Don't choose the Robotics Club."

Wendy: "You should choose both the Theater Club and the Yearbook Club" and "Don't choose the Choir."

Helen: "Choose at least one from the Robotics Club and the Rock Band Club" and "Don't choose the Theater Club."

Later Christina signed up for 3 clubs and told her friends that she took exactly one suggestion from each of them. What is the sum of 3 numbers for the clubs that Christina chose?

- (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

27. Cameo has three buckets labeled A, B, and C that have capacities of 21 liters, 50 liters, and 60 liters, respectively. She fills half of bucket A with coffee and half of bucket B with milk. She then empties the contents of bucket A and bucket B into bucket C and fills bucket C the rest of the way up with milk and stirs it around. After stirring bucket C thoroughly, she pours 50 liters of the solution in bucket C into bucket B. In the end, what is the absolute difference between the number of liters of milk and the number of liters of coffee in bucket B?

- (A) 8.5 (B) 10 (C) 29 (D) 32.5 (E) 35

28. Atticus likes to play with numbers. One day, he writes all the integers from 1 to 2022 on the whiteboard. Then, he repeatedly chooses two numbers on the whiteboard, erases them, and replaces them by their sum or product. For example, Atticus might erase 3 and 5 and then write 15 on the whiteboard. Later Atticus notices that all the remaining numbers on the whiteboard are even. Denote X as the maximum possible number of integers that remain on the whiteboard and Y as the minimum. What is the value of $X - Y$?

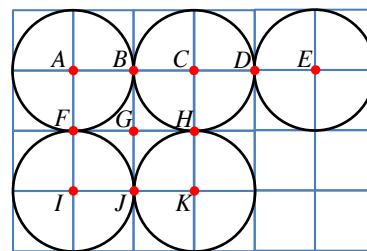
- (A) 1010 (B) 1011 (C) 1514 (D) 1515 (E) 1516

29. Kevin has a weird calculator. For any input number b , the calculator will return the value of $\frac{b-1}{b}$. For example, if Kevin inputs 2 and presses the button, the calculator will return $\frac{1}{2}$. If he pressed the button again, the calculator will return -1 , because $\frac{\frac{1}{2}-1}{\frac{1}{2}} = -1$. One day Kevin inputs a number and presses the button 20 times. He records all 21 numbers in order, including the first number he inputted and the 20 numbers returned by the calculator. He finds that the product of the last 2 numbers he recorded was 6. What is the 11th number he recorded?

- (A) -6 (B) $-\frac{1}{6}$ (C) 3 (D) 6 (E) 7

30. 5 circles of radius 1 are placed in a 4 by 6 grid (formed by 24 unit squares). Some vertices of the unit squares are colored red and are labeled A, B, ..., K. We call a straight line *perfect* if it passes through at least two red points and divides the 5 circles into two parts with equal areas. Which of the following points does not belong to any perfect lines?

- (A) A (B) E (C) F (D) J (E) K



The beauty of mathematics only shows itself to more patient followers.
 ----- Maryam Mirzakhani (The first woman mathematician to be honored with the Fields Medal)